Exercise 5 - Computational Models - Spring 2014

1. Describe an algorithms that decides $\{\langle M\ , w\rangle \mid M$ is a PDA that accepts $w\}$. Note that merely simulating a PDA using a NTM will not work - explain why.

2. For each of the following languages, find a minimal (relative to inclusion) class it belongs to (if any) between R/RE/coRE/not in RE∪co-RE. Prove your answer.

   (a) $EQ_{LBA} = \{\langle A_1, A_2\rangle \mid A_1, A_2$ are LBAs and $L(A_1) = L(A_2)\}$

   (b) $L = \{\langle M\rangle \mid \exists x\ s.t.\ M$ halt on $x\}$

   (c) $L = \{\langle M\rangle \mid$ in the computation of $M(\varepsilon)$ the head moves left at most 2014 times$\}$

   (d) $L = \{\langle M\rangle \mid$ in the computation of $M(\varepsilon)$ the head **never** moves three times **in a row** left$\}$

   (e) $L = \{\langle M\rangle \mid L(M) \subseteq L(1(0 \cup 1)^*)\}$

   (f) $L = \{\langle M\rangle \mid L(M) = L(1(0 \cup 1)^*)\}$

3. For $L \subseteq \Sigma^*$ let $A(L) = \{0w \mid w \in L\} \cup \{1w \mid w \notin L\}$

   - Prove that if $L \notin R$ then $A(L) \notin RE$
   - Prove or contradict: $\forall L \subseteq \Sigma^*$ if $L \leq_M \bar{L}$ then $L \in R$
     Hint: use $A(L)$ as $L \notin R$

4. Prove that the class $\mathcal{P}$ is closed under intersection, complement and concatenation.

5. Prove that the class $\mathcal{NP}$ is closed under union, intersection, concatenation and Kleene star.

6. Let $EXP = \bigcup_{c \geq 1} DTIME(2^{n^c})$. Prove that $\mathcal{P} \subseteq \mathcal{NP} \subseteq EXP$.